Math 115A A, Lecture 2
Real Analysis

Sample Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

Decide whether each set V with the laws of addition and scalar multiplication given is a vector space, and prove your answer.

(a) [5pts.] $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$ with $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and

$$c(a_1, a_2) = \begin{cases} (0,0) \text{ if } c = 0 \\ \left(ca_1, \frac{a_2}{c}\right) \text{ if } c \neq 0 \end{cases}$$

(b) [5pts.] V is the set of even functions $f : \mathbb{R} \to \mathbb{R}$. (An even function is one for which $f(t) = f(-t)$.)

Problem 2.

Let $S \subset V$ be a subset of a real vector space V.

(a) [5pts.] What does it mean to say that S is linearly independent?

(b) [5pts.] Prove that if S is linearly independent, and $v \notin S$, then $S \cup \{v\}$ is linearly dependent if and only if $v \in \mathrm{span}(S)$.

Problem 3.

Let $S \subset P_3(\mathbb{R})$ be the set $\{x^2 - 3x + 2, x^3 - 1, x^2 - 1, x^2 + 2x - 3\}$.

(a) [5pts.] Is S linearly independent or dependent?

(b) [5pts.] What is $\text{span}(S)$? [Hint: Consider factoring.]

Problem 4.

Let V be the subset of $\mathrm{Mat}_{3x3}(\mathbb{R})$ consisting of matrices of trace zero. (Recall that the trace of a matrix is the sum of its diagonal entries, so $\mathrm{tr}(A) = A_{11} + A_{22} + A_{33}$.)

(a) [5pts.] Prove that V is a subspace of $\mathrm{Mat}_{3\times 3}(\mathbb{R})$.

(b) [5pts.] Determine the dimension of V, and prove your answer.

Problem 5.

Let W_1 and W_2 be subspaces of a vector space V such that W_1 has dimension n and W_2 has dimension m, and $m \geq n$.

(a) [5pts.] Prove that $\dim(W_1 \cap W_2) \leq n$. [Hint: Produce a linearly independent set in W_1.]

(b) [5pts.] Prove that $\dim(W_1 + W_2) \leq m + n$. [Hint: Produce a generating set in $W_1 + W_2$.]